

Math 115

Fall 2017

Lecture 16



Ch. 4

Exponential Rules:

$$1) x^n = \underbrace{x \cdot x \cdot x \cdot x \cdots x}_{n \text{ times}}$$

$$2) x^1 = x$$

$$3) x^0 = 1; x \neq 0$$

$$4) x^m \cdot x^n = x^{m+n}$$

$$5) (x^m)^n = x^{m \cdot n}$$

$$6) (xy)^n = x^n y^n$$

$$7) \frac{x^m}{x^n} = x^{m-n}$$

$$8) \left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$$

$$9) x^{-n} = \frac{1}{x^n}$$

$$10) \frac{x^{-n}}{y^m} = \frac{y^m}{x^n}$$

$$11) \left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n$$

Simplify:

$$5^1 - 10^0 = 5 - 1 = \boxed{4}$$

$$\left(-\frac{3}{4}\right)^0 - \left(\frac{1}{2}\right)^{-1} = 1 - \left(\frac{2}{1}\right)^1 = 1 - 2^1$$

$$= 1 - 2 = \boxed{-1}$$

Expand: $(-3x^2)^4 = \underbrace{(-3x^2) \cdot (-3x^2) \cdot (-3x^2) \cdot (-3x^2)}_{4 \text{ times}}$

Write using exponent:

$$\frac{4}{x} \cdot \frac{4}{x} \cdot \frac{4}{x} \cdot \frac{4}{x} \cdot \frac{4}{x} = \left(\frac{4}{x}\right)^5$$

Simplify :

$$1) x^8 \cdot x^{10} = x^{8+10}$$

$$= \boxed{x^{18}}$$

$$2) (x^8)^{10} = x^{8 \cdot 10}$$

$$= \boxed{x^{80}}$$

$$3) (x^7)^3 \cdot x^9$$

$$= x^{7 \cdot 3} \cdot x^9$$

$$= x^{21} \cdot x^9 = \boxed{x^{30}}$$

$$4) (x^{10})^2 \cdot (x^5)^4$$

$$= x^{20} \cdot x^{20}$$

$$= \boxed{x^{40}}$$

$$5) (-5x^6)^3 = (-5)^3 (x^6)^3 \\ = \boxed{-125 x^{18}}$$

$$6) (2x^8y^3)^4 \\ = 2^4 (x^8)^4 (y^3)^4 \\ = \boxed{16 x^{32} y^{12}}$$

$$7) \frac{x^{12}}{x^3} = x^{12-3} \\ = \boxed{x^9}$$

$$8) \frac{(x^6)^4}{(x^3)^5} \\ = \frac{x^{24}}{x^{15}} \\ = x^{24-15} = \boxed{x^9}$$

Simplify (No negative exponent)

$$1) x^{-4} = \boxed{\frac{1}{x^4}}$$

$$2) 3^{-1} = \frac{1}{3^1} = \boxed{\frac{1}{3}}$$

$$3) (x^{-2})^5 = x^{-10} \\ = \boxed{\frac{1}{x^{10}}}$$

$$4) (x^{-3})^{-4} \cdot x^{-13} \\ = x^{12} \cdot x^{-13} \\ = x^{-1} = \frac{1}{x^1} = \boxed{\frac{1}{x}}$$

Simplify

$$1) \left(\frac{3x}{y^4} \right)^2$$

$$= \frac{3^2 x^2}{(y^4)^2} = \boxed{\frac{9x^2}{y^8}}$$

$$2) \left(\frac{x^3}{2y^5} \right)^{-3} = \left(\frac{2y^5}{x^3} \right)^3$$

$$= \frac{2^3 (y^5)^3}{(x^3)^3}$$

$$= \boxed{\frac{8y^{15}}{x^9}}$$

Simplify

$$\frac{-\cancel{24}^3 x^{12} y^{18}}{\cancel{16}^2 x^{10} y^8} = - \frac{3 x^2 y^{10}}{2}$$

Reduce
by 8

$$= \boxed{-\frac{3}{2} x^2 y^{10}}$$

$$x^{12-10} = x^2$$

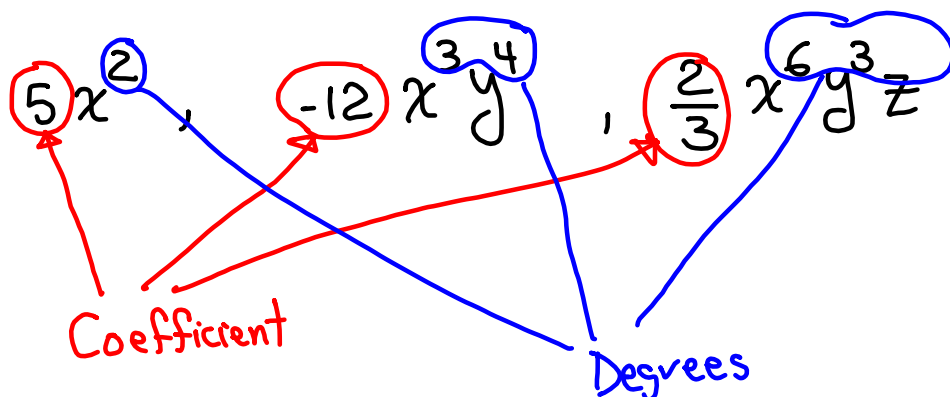
$$y^{18-8} = y^{10}$$

Simplify

$$\frac{-50 x^{12} y^{-6}}{15 x^{-8} y^{10}} =$$

$$\frac{-\cancel{50}^{10} x^{12} x^8}{\cancel{15}^3 y^{10} y^6} = \boxed{\frac{-10 x^{20}}{3 y^{16}}}$$

Monomial: # Variables ^{whole #}



$2017x^8$ Coef. 2017, Deg. 8

$-100x^2y^8$ Coef. = -100, Deg. $2+8=10$

A monomial without any variable is called constant. Constant has degree 0.

85 \rightarrow Monomial \rightarrow constant \rightarrow Deg. = 0

Monomial	Coef.	Degree
$25x^6$	25	6
$-32xy$	-32	$1+1=2$
$123xy^2z^3$	123	$1+2+3=6$
100	Constant	0

Binomial: + or - of two monomials

$$4x^2 + 10, -3x - 8, x^4y^2 - x^2y^4$$

$$\frac{2}{3}x - \frac{3}{5}y, x^2y^3z^5 + xyz$$

Deg. is the highest degree of the monomials.

Coef. of that monomial is called the leading Coef.

$$3x^{10} - 20x^2 \quad \text{Deg.} = 10 \quad \text{L.C.} = 3$$

$$4x^3y^7 - 12x^8y^5 \Rightarrow \text{Deg.} = 13$$

$$D = 10$$

$$D = 13$$

$$\text{L.C.} = -12$$

$$C = 4$$

$$C = -12$$

Monomial	D	C
$4x^3y^7$	10	4
$-12x^8y^5$	13	-12

$$D = 13$$

$$\text{L.C.} = -12$$

Find D & C of each monomial, then
give D & L.C. of the binomial

$$24x^6y^3 - 400xy$$

$$D = 9$$

$$L.C. = 24$$

Mon.	D	C
$24x^6y^3$	9	24
$-400xy$	2	-400

Trinomial: + or - of three monomials

$$x^2 - 5x + 7 \Rightarrow D = 2, L.C. = 1, \text{constant} = 7$$

$$4x^3 + 12x^2 - 100x \Rightarrow D = 3, L.C. = 4, \text{No constant}$$

$$x^6y^5 - 15x^3y^7 + xy$$

$$\text{Deg. } 11$$

$$L.C. = 1$$

Monomials	D	C
x^6y^5	11	1
$-15x^3y^7$	10	-15
xy	2	1

Monomial, binomial, trinomial are Special Polynomials.

Polynomial: + or - of monomials

$$7x^4 - 12x^3 + 25x^2 - 45$$

$$D=4, \text{ L.C.}=1, \text{ Const.}=-45$$

$$12x^4y^2 - 10x^6y^3 + \frac{2}{3}x^8y^5 + xy + 1$$

$D=6$ $D=9$ $D=13$ $D=2$ $D=0$

$$D=13, \text{ L.C.}=\frac{2}{3}, \text{ Constant}=1$$

Simplify

$$(4x^5)(12x^3) = \underbrace{4 \cdot 12}_{=48} \cdot x^5 \cdot x^3$$

Monomial

$$D=8$$

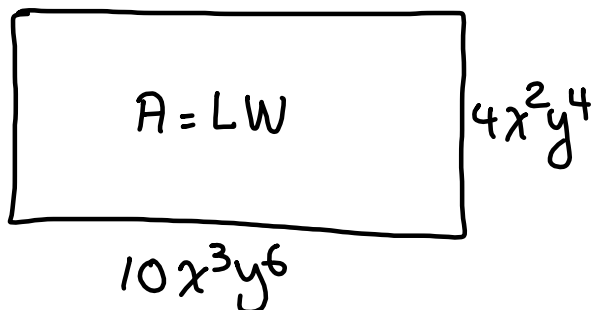
$$C=48$$

$$\frac{-60x^{12}y^8}{5x^3y^7} = -12x^9y^1 = -12x^9y$$

Monomial

$$D=9+1=10, C=-12$$

Find the area



$$A = LW$$

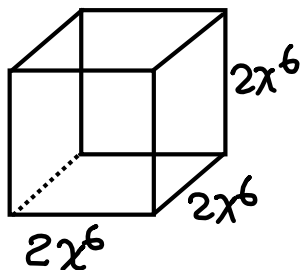
$$= (10x^3y^6)(4x^2y^4)$$

$$= 10 \cdot 4 x^3 x^2 y^6 y^4$$

$$= \boxed{40 x^5 y^{10}}$$

Monomial
 $D = 15$, Coef. 40

Find the Volume



$$V = S^3$$

$$V = (2x^6)^3$$

$$= 2^3 (x^6)^3$$

$$= \boxed{8 x^{18}}$$

Monomial

$$D = 18$$

$$C = 8$$

Recall $a(b+c) = ab + ac$

Distributive Prop.

$$2(x+3) = 2x + 6$$

$$-3(4x-5) = -12x + 15$$

$$5(2x^2 - 3x + 1) = 10x^2 - 15x + 5$$

$$4x(3x - 2) = 4x \cdot 3x - 4x \cdot 2$$

$$= \boxed{12x^2 - 8x}$$

$$3x^2(4x^2 - 5x + 6)$$

$$= 3x^2 \cdot 4x^2 - 3x^2 \cdot 5x + 3x^2 \cdot 6$$

$$= 12x^4 - 15x^3 + 18x^2$$

$$4x^3(5x^2 - 4x + 1)$$

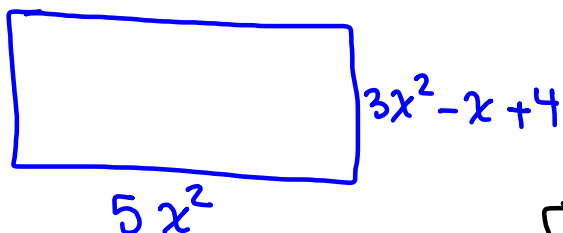
$$= 4x^3 \cdot 5x^2 - 4x^3 \cdot 4x + 4x^3 \cdot 1$$

$$= 20x^5 - 16x^4 + 4x^3$$

Trinomial
D=5
L.C.=20

Find the area

$$A = LW$$



$$A = 5x^2(3x^2 - x + 4)$$

$$= 5x^2 \cdot 3x^2 - 5x^2 \cdot x + 5x^2 \cdot 4$$

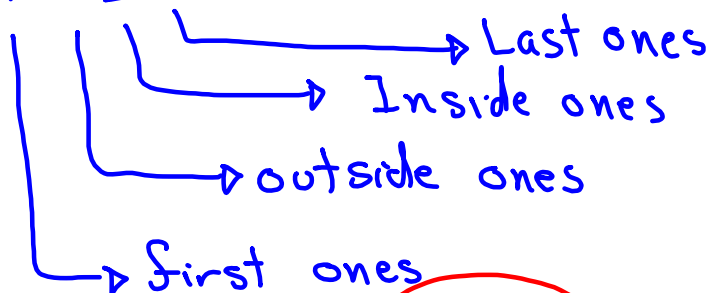
$$A = 15x^4 - 5x^3 + 20x^2$$

Trinomial

$$D = 4, L.C. = 15$$

No constant.

FOIL Method



Multiply $(x + 5)(x + 2) =$

Trinomial

$$D = 2$$

$$L.C. = 1$$

$$\text{Constant} = 10$$

$$x^2 + 2x + 5x + 10 =$$

$$x^2 + 7x + 10$$

Multiply by using FOIL method:

$$(2x + 4)(3x - 5)$$

$$= 2x \cdot 3x - 2x \cdot 5 + 4 \cdot 3x - 4 \cdot 5$$

$$= 6x^2 - 10x + 12x - 20$$

$$= \boxed{6x^2 + 2x - 20}$$

Trinomial

$$D=2$$

$$L.C.=6$$

$$\text{Constant} = -20$$

Multiply

$$(5x - 3)(4x - 5)$$

$$= 5x \cdot 4x - 5x \cdot 5 - 3 \cdot 4x + 3 \cdot 5$$

$$= 20x^2 - 25x - 12x + 15$$

$$= \boxed{20x^2 - 37x + 15}$$

$$D=2, L.C.=20$$

$$\text{Const. } 15$$

Trinomial

Multiply

$$(5x + 6)(5x - 6)$$

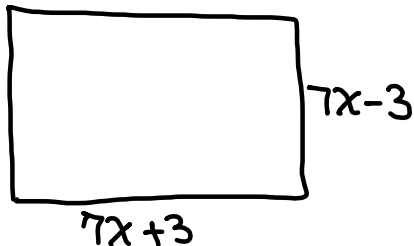
Conjugates

$$= 5x \cdot 5x - 5x \cdot 6 + 6 \cdot 5x - 6 \cdot 6$$

$$= 25x^2 - \cancel{30x} + \cancel{30x} - 36$$

$$= \boxed{25x^2 - 36}$$

Find the area



$$A = LW$$

$$= (7x+3)(7x-3)$$

Conjugates

$$= 49x^2 - \cancel{21x} + \cancel{21x} - 9$$

$$= 49x^2 - 9$$

Use FOIL method to multiply

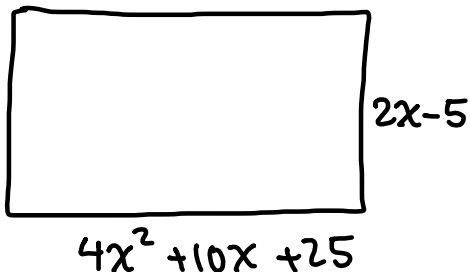
$$(x^1 + 3)(x^2 - 3x + 9)$$

$$= x^3 - \cancel{3x^2} + \cancel{9x} + \cancel{3x^2} - \cancel{9x} + 27$$

$$= \boxed{x^3 + 27}$$

binomial D=3 L.C. 1
Const 27

Find the area



$$A = LW$$

$$A = (2x-5)(4x^2 + 10x + 25)$$

$$= 8x^3 + \cancel{20x^2} + \cancel{50x} - \cancel{20x^2} - \cancel{50x} - 125$$

$$= \boxed{8x^3 - 125}$$

Binomial

D=3, L.C. 8

Const -125